BOOK 0
ARITHMETIC
BOOK 1
ALGEBRA I
BOOK 2
ALGEBRA II
BOOK 3
TRIGONOMETRY
BOOK 4
PLANE GEOMETRY
BOOK 5
SOLID GEOMETRYT
BOOK 6
ANALYTIC GEOMETRY
BOOK 7
CALCULUS (LIMITS)
BOOK 8
CALCULUS (DERIVATIVES)
BOOK 9
CALCULUS (INTEGRALS)
MathVantage

MathVantage Online courses Availability

The material is completely self-explanatory, but MathVantage incentivizes its students to discuss its contents with friends, parents, and professors. This collection is an amazing affordable opportunity to achieve a solid math background in a short time. (Every lesson or exam takes, on average only 1 or 2 hours to be completed)

Book 0: Arithmetic

Book 1: Algebra I

Book 2: Algebra II

Book 3: Trigonometry

Book 4: Plane Geometry

Book 5: Solid Geometry

Book 6: Analytic Geometry

Book 7: Calculus - Limits

Book 8: Calculus - Derivatives

Book 9: Calculus - Integrals

Book 10: Math - SSAT/ACT - College Prospective (Not Available yet)

Book 11: Statistics and Probability (Not Available yet).

Lesson 8 Numerical Expressions

I. Introduction

A **numerical expression** is a set of numbers that have been written together by utilizing arithmetic operators addition, subtraction, multiplication, and division.

Law of Signs

Addition and Subtraction

Rule 1: Numbers with same sign, add the numbers (magnitudes) and keep the sign.

Rule 2: Numbers with different sign, subtract the numbers (magnitudes) and keep the sign of the larger one.

Examples

Rule 1: Same sign.

Add magnitudes

$$2 + 3 = +\,(2 + 3) = +\,5$$

$$-2 - 3 = -\,(2 + 3) = -\,5$$

Keep the sign

Rule 2: Different sign.

Subtract magnitudes

$$-2 + 3 = +\,(3 - 2) = +\,1$$

$$+2 - 3 = -\,(3 - 2) = -\,1$$

Use the large number sign

Multiplication and Subtraction

Rule 1: Numbers with same sign, the result will be positive.

Rule 2: Numbers with different sign, the result will be negative.

Examples

Rule 1: Same sign.

a. $(+4) \text{ x } (+2) = +\,8$

b. $\dfrac{(+4)}{(+2)} = +\,2$

$+\ + \Rightarrow +$

Same sign (positive)

c. $(-4) \text{ x } (-2) = +\,8$

d. $\dfrac{(-4)}{(-2)} = +\,2$

$-\ - \Rightarrow +$

Rule 2: Different sign.

a. $(+4) \text{ x } (-2) = -\,8$

b. $\dfrac{(+4)}{(-2)} = -\,2$

$+\ - \Rightarrow -$

Different sign (negative)

c. $(-4) \text{ x } (+2) = -\,8$

d. $\dfrac{(-4)}{(+2)} = -\,2$

$-\ + \Rightarrow -$

II. PEMDAS

In math, **order of operations** are the rules that state the sequence in which the multiple operations in an expression should be solved.

A way to remember the order of the operations is **PEMDAS**, where in each letter stands for a mathematical operation.

Order of Operations (PEMDAS)	
P	**Parentheses**
E	**Expoent**
M	* **Multiplication**
D	**Division**
A	* **Addition**
S	**Subtraction**

* **Note1: Multiplication / Division** have the same priority. If you have both multiplication and division, do the operations one by one in the order from left to right.

* **Note2: Addiction / Subtraction** have the same priority. If you have both addition and subtraction, do the operations one by one in the order from left to right.

Example:

Solve: $10 \div 5 \text{ x } (10 \div 5) \text{ x } 10 \div 5$

$10 \div 5 \text{ x } (10 \div 5) \text{ x } 10 \div 5$

$10 \div 5 \text{ x } 2 \text{ x } 10 \div 5$

$2 \text{ x } 2 \text{ x } 10 \div 5$

$4 \text{ x } 10 \div 5$

$40 \div 5$

8

Exercises

1. Solve:

a. $9 - 6 \div 3 \text{ x } 5$

$9 - 2 \text{ x } 5$

$9 - 10$

-1

b. $(9 - 6) \div 3 \text{ x } 5$

$3 \div 3 \text{ x } 5$

$1 \text{x } 5$

5

c. $1 \text{ x } 2 + 3 - 4 \text{ x } 5 \div 6$

$2 + 3 - 4 \text{ x } 5 \div 6$

$2 + 3 - 20 \div 6$

$2 + 3 - \frac{10}{3}$

$\frac{15}{3} - \frac{10}{3}$

$\frac{5}{3}$

d. $6 + (-16) \div 2 \text{ x } (-2)$

$6 + (-8) \text{ x } (-2)$

$6 + 16$

22

Lesson 5 Quadratic Equations

I. Introduction

Quadratic Equations are in the following form:

$$ax^2 + bx + c = 0; \quad a \neq 0$$

Notation:

x : Variable

a, b, c : Constants

x_1, x_2 : Roots (Solutions)

$S = x_1 + x_2$ (Sum of Roots)

$P = x_1 \cdot x_2$ (Product of roots)

$\Delta = b^2 - 4ac$ (Discriminant)

Formulas:

$$ax^2 + bx + c = 0; \quad a \neq 0$$

$$S = -\frac{b}{a} \quad \text{and} \quad P = \frac{c}{a}$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}; \quad \Delta = b^2 - 4ac$$

$$ax^2 + bx + c = a\left(x - x_1\right)\left(x - x_2\right)$$

The Quadratic equation has:

- no solution if $\Delta < 0$.
- one distinct solution if $\Delta = 0$.
- two distinct solutions if $\Delta > 0$

Let $S = x_1 + x_2$ and $P = x_1 \cdot x_2$, then one equation with the solutions x_1, x_2 are:

$$x^2 - Sx + P = 0$$ (Professor formula)

Why can S and P be considered as the soul of the quadratic equation?

Procedure to find integer solutions for quadratic equations quickly:

1. Calculate S and P.

2. List all the positive factor pairs of P.

   Note: Start the first column with 1 and P
   Add new factor pairs in increasing order
   until a number is repeated.

3. Determine the signs of the factors:
   The second column has the same sign as S.
   If $P > 0$, both columns have the same sign.
   If $P < 0$, the first column has the opposite sign of the second column.

4. Select the factor pair with the sum equal to S.

   Example: Let $P = -12$ and $S = 1$, then:

-1	$+12$	$\Rightarrow$ Sum $= 11$	
-2	$+6$	$\Rightarrow$ Sum $= 4$	
-3	$+4$	$\Rightarrow$ Sum $= 1$	Ok

PS: S and P can be considered as the soul of the quadratic equation because if S and P didn't solve it very fast, they could be used to check easily their roots x_1 and x_2.

2. Let x_1 and x_2 be the solutions to $x^2 - 10x + 10 = 0$. Find:

a. $x_1 + x_2 = -\frac{b}{a} = 10$

b. $x_1 \cdot x_2 = \frac{c}{a} = 10$

c. $(x_1)^2 + (x_2)^2 =$

$\left(x_1 + x_2\right)^2 = (x_1)^2 + 2x_1x_2 + (x_2)^2$

$(x_1)^2 + (x_2)^2 = S^2 - 2P$

$= 10^2 - 2(10)$

$= 80$

d. $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{(x_1)^2 + (x_2)^2}{x_1x_2}$

$= \frac{80}{10} = 8$

* e. $(x_1)^3 + (x_2)^3 = \left(x_1 + x_2\right)\left((x_1)^2 - x_1x_2 + (x_2)^2\right)$

$= S(S^2 - 2P - P)$

$= S(S^2 - 3P)$

$= S^3 - 3SP$

$= 10^3 - 3 \cdot 10 \cdot 10$

$= 1{,}000 - 300$

$= 700$

Lesson 5 Homework

1. Solve

a. $x^2 - 10x + 24 = 0$

b. $x^2 - 6x + 6 = 0$

2. Simplify

a. $\frac{x^2 - 2x - 3}{x + 1} = 0$

b. $\frac{x^4 - 5x^2 + 4}{x^2 - 4} = 0$

3. Let x_1 and x_2 be the solution to

$x^2 - x - 1 = 0$. Find:

a. $x_1 + x_2 =$

b. $x_1 \cdot x_2 =$

c. $(x_1)^2 + (x_2)^2 =$

d. $\frac{x_1}{x_2} + \frac{x_2}{x_1} =$

* e. $(x_1)^3 + (x_2)^3 =$

4. Solve

a. $\sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}} = 2; x \geq 0$

Lesson 4 Quadratic Functions

I. Introduction

Quadratic function is any function in the form:

$$y = ax^2 + bx + c; \quad a \neq 0$$

1. Factoring Form:

$$y = a\left(x - x_1\right)\left(x - x_2\right); \ x_1 \text{ and } x_2 \text{ (Roots)}$$

2. Graph of the quadratic function (Parabola).

$$y = ax^2 + bx + c$$

Procedure:

a. Find the roots of the quadratic equation.

$ax^2 + bx + c = 0$, using the sum and product technique.

$$S = -\frac{b}{a} \text{ and } P = \frac{c}{a}$$

b. If the roots are not found then calculate:

$$\Delta = b^2 - 4ac \text{ and } x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

If $\Delta = 0 \Rightarrow$ Two distinct real roots

If $\Delta > 0 \Rightarrow$ One real root

If $\Delta < 0 \Rightarrow$ No real roots

c. Find the vertex of the parabola.

Method 1: Formulas:

$$x_V = \frac{-b}{2a} \text{ and } y_v = \frac{-\Delta}{4a}$$

Method 2 : Symmetry.

$$x_v = \frac{x_1 + x_2}{2}, \text{ (average of } x_1 \text{ and } x_2\text{)}$$

$$y_V = f\left(x_v\right) = a\left(x_v\right)^2 + b\left(x_v\right) + c$$

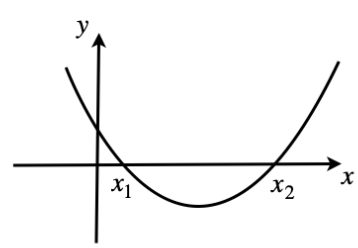
d. Find the x and y intercepts.

x intercepts are $x_1, x_2 \ldots (y = 0)$

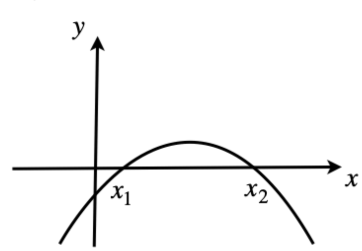
y intercept is "c." $(x = 0)$

e. Determining the concavity:

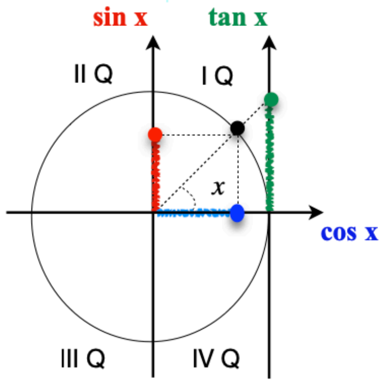
a $> 0 \Rightarrow$ Upward



a $< 0 \Rightarrow$ Downward



IV. Trigonometric Ball



V. Trigonometric Table

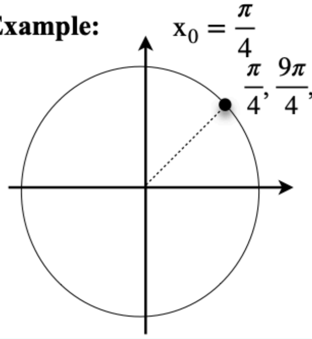
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin x$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos x$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan x$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

General Solution:

$$\underset{\text{Angle}}{x} = \underset{\text{Initial}}{x_0} + \underset{\text{Period}}{2\pi k}, \; k \in \mathbb{Z}$$

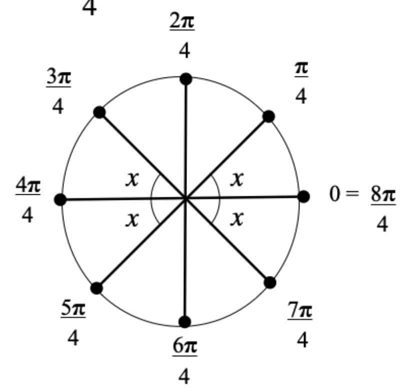
Example: $x_0 = \frac{\pi}{4}$

$\frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, -\frac{7\pi}{4}, -\frac{15\pi}{4}, \ldots$



Given an angle in the I Quadrant, we can easily get the angles in the other quadrants.

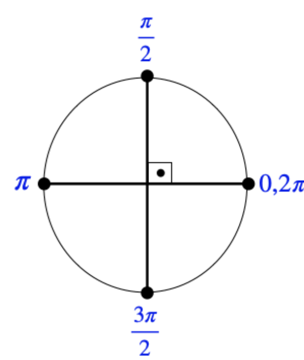
Example: $x = \frac{\pi}{4}$



V. Examples

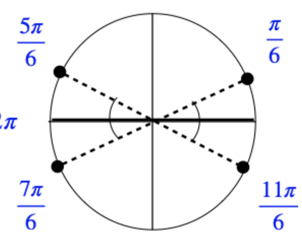
1. Find the angles and the general solution for the trigonometric balls.

a. $x = 90^\circ$



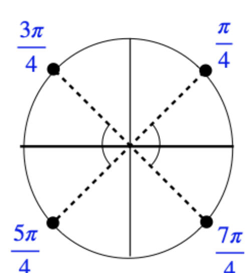
$x = \frac{\pi}{2}k, \; k \in \mathbb{Z}$

b. $x = 30^\circ$



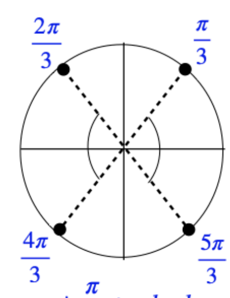
$x = \pm\frac{\pi}{6} + \pi k, \; k \in \mathbb{Z}$

c. $x = 45^\circ$



$x = \frac{\pi}{4} + \frac{\pi}{2}k, \; k \in \mathbb{Z}$

d. $x = 60^\circ$



$x = \pm\frac{\pi}{3} + \pi k, \; k \in \mathbb{Z}$

Lesson 5 Right Triangle

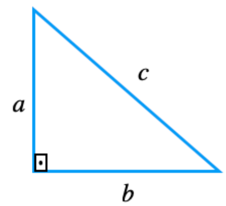
I. Introduction

Right triangles are triangles in which one of the interior angles is 90°.

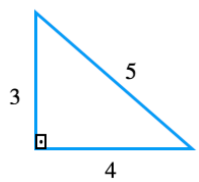
Pythagorean theorem

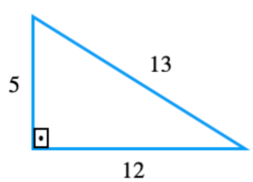
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. This relationship is represented by the formula:

$$a^2 + b^2 = c^2$$

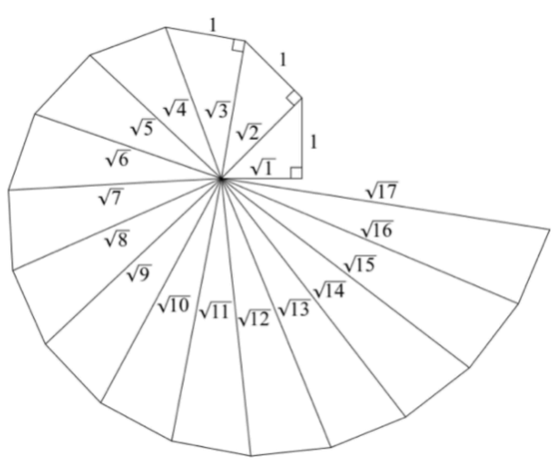


The most famous Pythagorean triangles are:





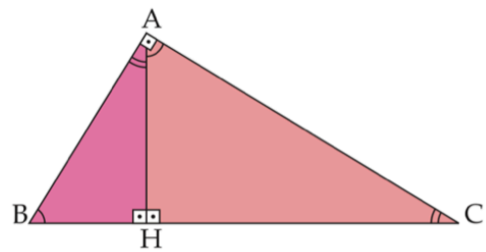
A **Square Root Spiral** is a series of right triangles arranged in a spiral configuration such that the hypotenuse of one right triangle is a leg of the next right triangle.



III. Exercises

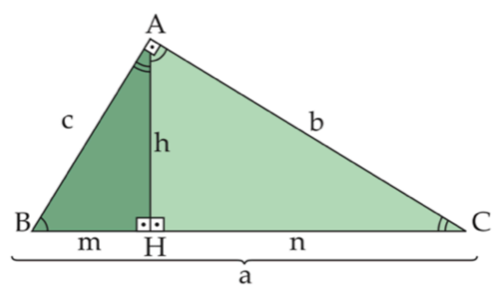
Metric Relations in a Right Triangle

In every right triangle, the height relative to the hypotenuse determines two triangles similar to the first and similar to each other.



$$\Delta ABC \sim \Delta HBA \sim \Delta HAC$$

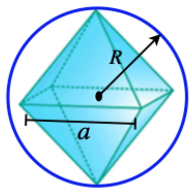
These three similar triangles derive the following five important formulas in the right triangle.



$$c^2 = a \,.\, m$$
$$b^2 = a \,.\, n$$
$$h^2 = m \,.\, n$$
$$b \,.\, c = h \,.\, a$$
$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$$

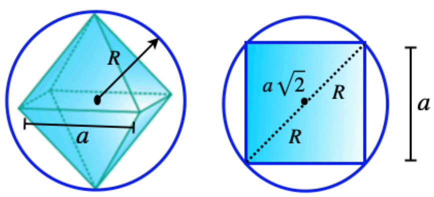
II. Examples

1. Find the radius r of a circumscribed sphere to a regular octahedron with edge a.



Solution: $R = \dfrac{a\sqrt{2}}{2}$

Critical Section



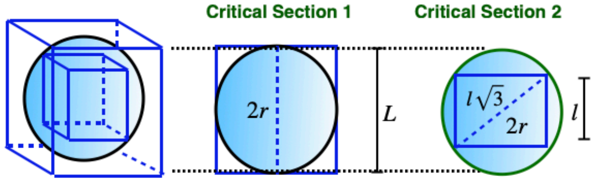
In the critical section, we have:

$2R = a\sqrt{2} \Rightarrow R = \dfrac{a\sqrt{2}}{2}$

2. Calculate the ratio between the volumes of the circumscribed and the inscribed cubes on a sphere with radius r.

Solution: $\dfrac{V}{v} = 3\sqrt{3}$

Critical Section 1 Critical Section 2



In section 1, $L = 2r$

In section 2, $l\sqrt{3} = 2r \Rightarrow l = \dfrac{2\sqrt{3}}{3}r$

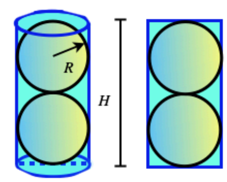
$K = \dfrac{L}{l} \Rightarrow K = \dfrac{2r}{\left(\frac{2\sqrt{3}}{3}r\right)} \Rightarrow K = \sqrt{3}$

$\dfrac{V}{v} = K^3 \Rightarrow \dfrac{V}{v} = (\sqrt{3})^3 \Rightarrow \dfrac{V}{v} = 3\sqrt{3}$

III. Exercises

1. In a 12 cm high cylindrical container, we place two spheres, one over the other, in such a way that these spheres touch the bases of the cylinder and its lateral surface. Determine the difference between the volume (cm^3) of the cylinder and the volume of the two spheres.

Critical Section



a) 4π b) 12π c) 24π d) 36π e) 48π

Solution: d

$H = 12$ cm

$H = 4R \Rightarrow R = \dfrac{H}{4} \Rightarrow R = \dfrac{(12)}{4} \Rightarrow R = 3$ cm

$V = V_{cylinder} - 2V_{sphere}$

$V = \pi R^2 H - 2\left(\dfrac{4\pi R^2 H}{3}\right)$

$V = \pi(3)^2(12) - 2\left(\dfrac{4\pi(3)^3}{3}\right)$

$V = 36\pi$ cm^3

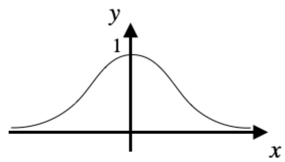
Lesson 4 Special Limits

I. Special Limits

The two special limits are:

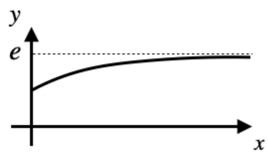
$$y = \frac{\sin x}{x}$$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$



$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\boxed{\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e}$$



II. Examples

1. Find the following limits given that:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

a. $\lim_{x\to 0} \frac{5x}{\sin x} = 5 \lim_{x\to 0} \frac{x}{\sin x}$

$$= 5 \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}$$

$$= 5 \left[\frac{1}{\lim_{x\to 0} \frac{\sin x}{x}}\right] = \frac{5}{1} = 5$$

b. $\lim_{x\to 0} \frac{\sin 5x}{x} =$

Let $\quad y = 5x \left\{ \; y \to 0 \right.$

$$= \lim_{y\to 0} \frac{\sin y}{\frac{y}{5}}$$

$$= \lim_{y\to 0} \frac{5 \sin y}{y}$$

$$= 5 \lim_{y\to 0} \cancelto{1}{\frac{\sin y}{y}} = 5$$

2. Find the following limit given that:

$$\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x = \lim_{x\to\infty} \left(1 + \frac{1}{\frac{x}{2}}\right)^x$$

Let $y = \frac{x}{2} \Rightarrow x = 2y \Longrightarrow \begin{cases} x \to \infty \\ y \to \infty \end{cases}$

$$\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x = \lim_{y\to\infty} \left(1 + \frac{1}{y}\right)^{2y}$$

$$\lim_{y\to\infty} \left[\left(1 + \frac{1}{y}\right)^y\right]^2$$

$$\left[\lim_{y\to\infty} \left(1 + \frac{1}{y}\right)^y\right]^2 = e^2$$

III.Exercises

1. Find the following limits given that:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

a. $\lim_{x\to 0} \frac{\sin x - x}{x} = \lim_{x\to 0} \frac{\sin x}{x} - \lim_{x\to 0} \frac{x}{x}$

$$= 1 - 1$$

$$= 0$$

b. $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\cos x}$

$$= 1 \cdot 1$$

$$= 1$$

c. $\lim_{x\to 0} \frac{\sin 2x}{x} =$

Let $y = 2x \Rightarrow x = \frac{y}{2} \Rightarrow \begin{cases} x \to 0 \\ y \to 0 \end{cases}$

$$\lim_{y\to 0} \frac{\sin y}{\frac{y}{2}} = \lim_{y\to 0} 2 \frac{\sin y}{y}$$

$$= 2 \lim_{y<0} \cancelto{1}{\frac{\sin y}{y}} = 2$$

2. Find the tangent line of the curve $x^3 + 2y^2 = 10$ at the point $P(2,1)$.

Hint: Tangent line: $y - y_0 = \frac{dy}{dx}(x - x_0)$

Using implicit derivative at $P(2,1)$ we have:

$3x^2 + 4y \cdot y' = 0$

$3 \cdot (2)^2 + 4 \cdot (1) \cdot y' = 0$

$12 + 4y' = 0$

$y' = -3$

$\frac{dy}{dx} = -3$

The tangent line of the curve at $P(2,1)$ are:

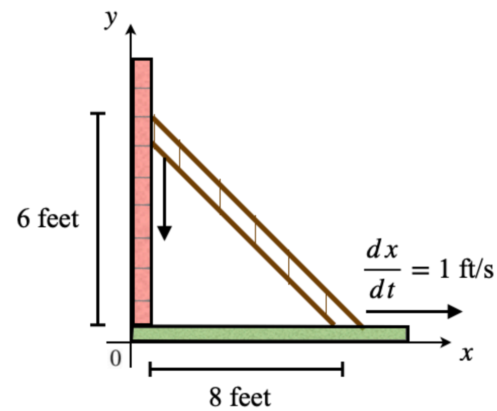
$y - y_o = \frac{dy}{dx}(x - x_o)$

$y - 1 = -3(x - 2)$

$y - 1 = -3x + 6$

$y = -3x + 7.$

*3. A ladder 10 ft long is resting against a wall. If the bottom of the ladder is sliding away from the wall at a rate of 1 ft/s, how fast is the top of the ladder moving down when the bottom of the ladder is 8 ft from the wall?



By the Pythagorean theorem:

$x^2 + y^2 = 100$

The implicit derivative of the equation $x^2 + y^2 = 100$ at the point $P(8,6)$ is:

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

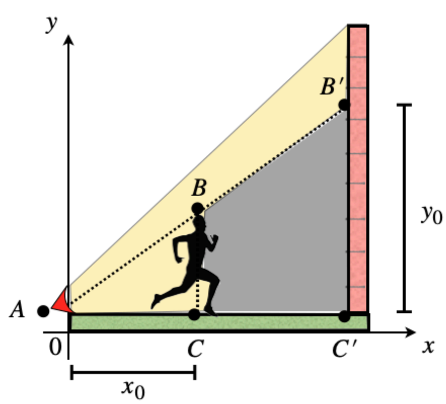
$2 \cdot (8) \cdot 1 + 2 \cdot (6) \cdot y' = 0$

$16 + 12y' = 0$

$y' = -\frac{4}{3}$

$\frac{dy}{dx} = -\frac{4}{3}$ ft/s

*Extra. A 6 ft tall prisoner is attempting to escape from the minimum security prison in North Bay. He runs in a straight line towards the prison wall at a speed of 5ft/s. The guards shine a spotlight, located on the ground 100 ft from the wall, on the prisoner as he begins to run. At what rate $\left(\dfrac{dy}{dx}\right)$ does his shadow on the prison wall decreases when he is 50 ft away from the wall?



$\triangle ABC \sim \triangle AB'C' \Rightarrow \dfrac{\overline{AC}}{\overline{BC}} = \dfrac{\overline{AC'}}{\overline{B'C'}}$

$\dfrac{x}{6} = \dfrac{100}{y} \Rightarrow xy = 600$

For $x_0 = 50$ ft, we have:

$50y_0 = 600 \Rightarrow y_0 = 12$ ft.

The implicit derivative of the equation $xy = 600$ at the point $P(x_0,y_0)$ is:

$\dfrac{dx}{dt}y + x\dfrac{dy}{dt} = 0$

$\dfrac{dx}{dt}y_0 + x_0\dfrac{dy}{dt} = 0$

$5 \cdot 12 + (50)\dfrac{dy}{dt} = 0$

$\dfrac{dy}{dt} = -1.2$ ft/s

Lesson 4 Homework

1. Find $\dfrac{dy}{dx}$ implicitly, given the following equations:

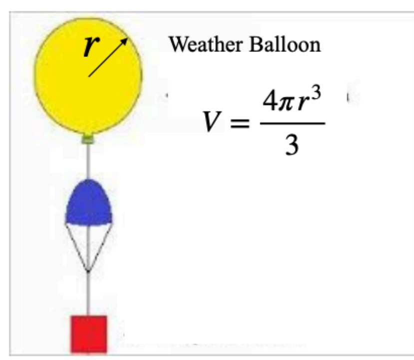
a. $y = x^2 + 2 = 0$

b. $2y = x^2 + 2 + y$

2. Use implicit derivative to determine the slope $\left(\dfrac{dy}{dx}\right)$ of the ellipse $x^2 + 4y^2 = 40$ at the point $P(2,3)$.

3. Find the equation of the tangent line to the curve $x^4 + y^4 = 2xy$ at $P(1,1)$.

*4. A large yellow weather balloon is being deflated and its volume V is decreasing at a rate of change $\left(\dfrac{dV}{dt}\right)$ of 24 ft^3/h. Find the rate of change of the radius $\left(\dfrac{dr}{dt}\right)$ when the radius is 2 ft.



Lesson 1 Integrals

I. Introduction

There are two fundamental problems of calculus. The first one is to find the slope of a curve at a point (Derivatives), and the second is to find the area of a region under a curve (Integral). These problems are quite easy in high school when the function is linear.

Example - High School

The velocity of a car is given by the following function: $v(t) = 2t$

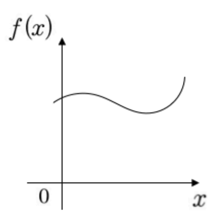
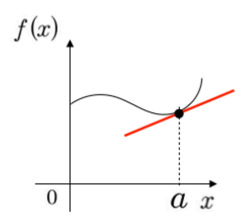
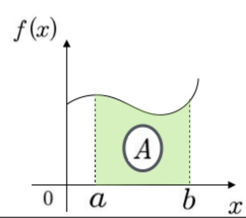
Acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0} = \frac{2-0}{1-0} = 2 \text{ m/s}^2$$

Displacement

$$A = \frac{b \cdot h}{2} = \frac{1 \cdot 2}{2} = 1 \text{ m}$$

	High School	College
Acceleration $\left(\frac{m/s}{s}\right)$	Slope $\left(m = \frac{\Delta v}{\Delta t}\right)$	Derivative
Displacement $\left(\frac{m}{s} \cdot s\right)$	Area $\left(A = \frac{b \cdot h}{2}\right)$	Integral

	Function	Derivative	Integral
Notation	$f(x)$	$f'(x)$	$\int_a^b f(x)dx$
Representation			
Meaning	$f(x)$ as a function of x	$f'(a)$ is the slope of the tangent line at $x = a$.	The area of the curve $f(x)$ on the interval [a, b].

Lesson 4 Integration by Parts

I. Introduction

The table of integrals and substitution are the first attempts to solve integrals. The integration by parts is a powerful technique to solve a much larger set of functions. There are two ways to do integration by parts: The standard method and the column method.

II. Standard Method

If u and v are differentiable functions then:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Sometimes the integral $\int v\ du$ is easier to solve than the integral: $\int u\ dv$.

Example: Solve $I = \int e^x x\ dx$

1. First Attempt: $I = \int \underbrace{e^x}_{u}\ \underbrace{x\ dx}_{dv}$

$$u = e^x \Rightarrow \frac{du}{dx} = e^x \Rightarrow du = e^x\,dx$$

$$dv = x\ dx \Rightarrow \int dv = \int x\ dx \Rightarrow v = \frac{x^2}{2}$$

Note: The constant "c" will be added only in the final answer.

Then: $\int u\ dv = uv - \int v\ du$

$$\int e^x x\,dx = e^x \frac{x^2}{2} - \int \underbrace{\frac{x^2}{2} e^x\,dx}$$

(More difficult Integral)

2. Second Attempt: $I = \int \underbrace{x}_{u}\ \underbrace{e^x\ dx}_{dv}$

$$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$$

$$dv = e^x dx \Rightarrow \int dv = \int e^x dx \Rightarrow v = e^x$$

Then $\int u\ dv = uv - \int v\ du$

$$\int x e^x dx = x e^x - \int \underbrace{e^x\,dx}$$

(Easier Integral)

$$\int x e^x\ dx = x e^x - e^x + c$$

Note: Add a constant "c" in the final answer.

III. Rule of Thumb

How Do You Choose the u?

(**L**et **P**atrick **T**eaching **E**asy)

L	P	T	E
↓	↓	↓	↓
L	P	T	E
O	O	R	X
G	L	I	P
	Y	G	O

Example:

$$\int e^x x\ dx \Rightarrow \int x e^x\ dx$$

$u = x$ (Polynomial)

$dv = e^x dx$ (Exponencial)

Note: Polynomial functions have a higher priority than exponential functions.

* Extra. $I = \int_0^{\frac{\pi}{4}} \sin x \cos x \; dx$

D		I
$\sin x$	$+$	$\cos x$
$\cos x$	$-$	$\sin x$

$$I = \sin^2 x - \int \sin x \cos x \; dx$$

$$I = \sin^2 x - I$$

$$2I = \sin^2 x$$

$$I = \frac{\sin^2 x}{2} + c$$

$$I = \left[\frac{\sin^2 x}{2}\right]_0^{\frac{\pi}{4}}$$

$$I = \frac{\sin^2\left(\frac{\pi}{4}\right)}{2} - \frac{\sin^2(0)}{2}$$

$$I = \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{2}$$

$$\boxed{I = \frac{1}{4}}$$

Lesson 4 Homework

1. Solve the following integrals:

a. $\int x \cos x \; dx =$

b. $\int \ln x \; dx =$

c. $\int x^2 \sin x \; dx =$

d. $\int e^x \sin x \; dx =$