PART 1: QUESTIONS

Name:________________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Limits - Exam 1 **Lessons: 27 - 29**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cell-phone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. Calculus is the most important math tool used in college. The meaning and notation of limits, derivatives and integrals is:

I. Limits means 'maximum or minimum' and the notation is $\lim f(x)$.

II. Derivatives means 'slope' and the notation is $\frac{df}{dx} = f'(x)$.

III. Integral means 'area' and the notation is $F(x) = \int f(x)dx$

a) Only I and II are correct
b) Only I and III are correct
c) Only II and II are correct
d) I, II, and III are correct
e) None of the above.

Solution: c

I. False. Limits means 'neighbor' and the notation is $\lim_{x \to a} f(x)$.

	Meaning	Notation
Limits	Neighbor	$\lim_{x \to a} f(x)$
Derivatives	Slope	$\frac{df}{dx} = f'(x)$
Integrals	Area	$F(x) = \int f(x)dx$

2. The notation $\lim_{x \to a} f(x) = L$ is:

a) the limit of x, as $f(x)$ approaches to a, is equal to L.
b) the limit of x, as $f(x)$ approaches to L, is equal to a.
c) the limit of $f(x)$, as x approaches to a, is equal to L.
d) the limit of $f(x)$, as x approaches to L, is equal to a.
e) None of the above.

Solution: c

We write $\lim_{x \to a} f(x) = L$ and say, the limit of $f(x)$, as x approaches to a, is equal to L.

3. The notation $\lim_{x \to a^-} f(x) = L$ is:

a) the right hand limit of f(x), so x approaches to "a" from the right, is equal to L.
b) the left hand limit of f(x), so x approaches to "a" from the left, is equal to L.
c) the limit of $f(x)$, as x approaches to L, is equal to a.
d) the limit of $f(x)$, as x approaches to a, is equal to L.
e) None of the above.

Solution: b

We write $\lim_{x \to a^-} f(x) = L$ and say, the left hand limit of f(x), so x approaches to "a" from the left, is equal to L.

4. The notation $\lim_{x \to a^+} f(x) = L$ is:

a) the limit of $f(x)$, as x approaches to L, is equal to a.
b) the left hand limit of f(x), so x approaches to "a" from the left, is equal to L.
c) the right hand limit of f(x), so x approaches to "a" from the right, is equal to L.
d) the limit of $f(x)$, as x approaches to a, is equal to L.
e) None of the above.

Solution: c

We write $\lim_{x \to a^+} f(x) = L$ and say, the right hand limit of f(x), so x approaches to "a" from the right, is equal to L.

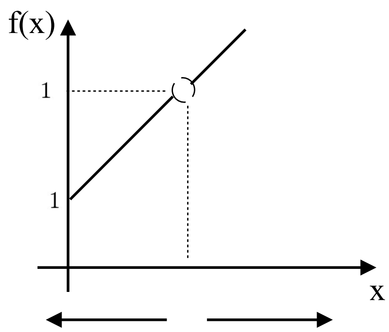
5. The notation $\lim_{x \to \infty} f(x) = L$ is:

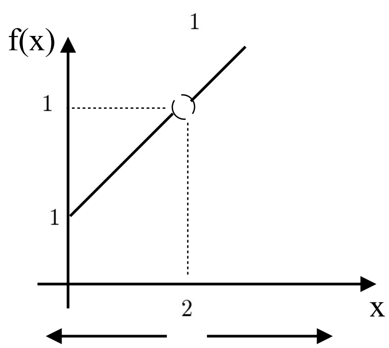
a) the limit of $f(x)$, as x approaches to ∞, is equal to $f(x)$.
b) the limit of $f(x)$, as x approaches to L, is equal to ∞.
c) the limit of ∞, as x approaches to L, is equal to $f(x)$.
d) the limit of ∞, as x approaches to ∞, is equal to $f(x)$.
e) None of the above.

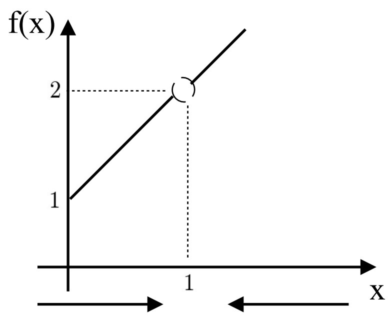
Solution: e

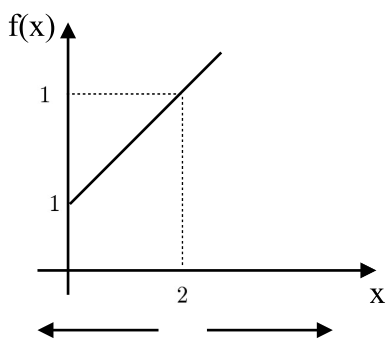
We write $\lim_{x\to\infty} f(x) = L$ and say, the limit of $f(x)$, as x approaches to ∞, is equal to L.

6. The best graph representation of $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$ is:

a)


b)


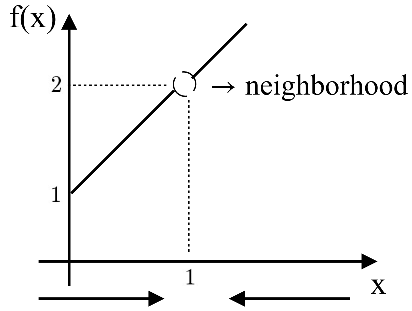
c)


d)


e) None of the above.

Solution: c

The best graph representation of $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$ is



$\lim_{x\to 1^-} \frac{x^2-1}{x-1} = 2$ $\lim_{x\to 1^+} \frac{x^2-1}{x-1} = 2$

7. Given:

I. Function $f(x)$ is continuous at $x = a$
$\iff \lim_{x\to a} f(x) \neq f(a)$.

II. If $\lim_{x\to a} g(x) = b$ and f is continuous at b
$\iff \lim_{x\to a} f[g(x)] = f\left[\lim_{x\to a} g(x)\right]$

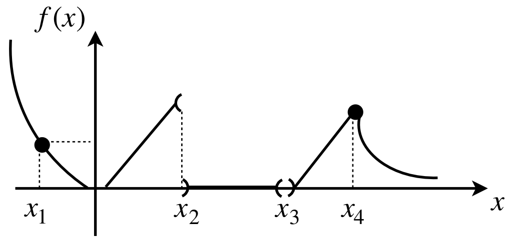
III. The limit $\lim_{x\to a} f(x) = L$ exists
$\iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$.

a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

Solution: c

I. False. Function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x\to a} f(x) = f(a)$.

8. Given the graph of $f(x)$.



I. $f(x)$ is continuous at $x = x_1$ and $x = x_4$.
II. Limit of $f(x)$ exists at $x = x_1$, $x = x_3$, and $x = x_4$.
III. Limit of $f(x)$ doesn't exists at $x = x_2$.

a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

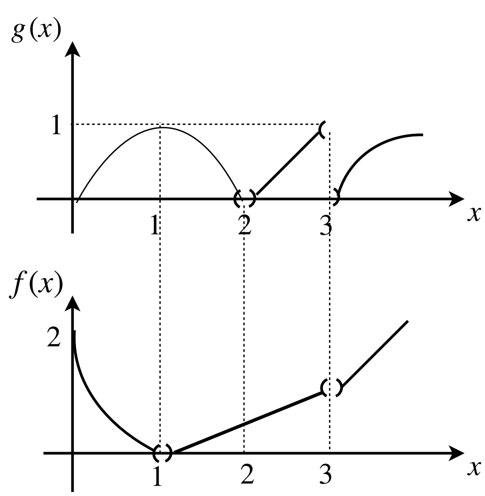
Solution: d

All alternatives are correct.

9. Given the graphs of $f(x)$ and $g(x)$ and the following limit property:

If $\lim_{x\to a} g(x) = b$ and f is continuous at b

$\iff \lim_{x\to a} f[g(x)] = f\left[\lim_{x\to a} g(x)\right]$.



Then,

I. $\lim_{x\to 1} f[g(x)] \neq f\left[\lim_{x\to 1} g(x)\right]$ because f is not continuous at 1.

II. $\lim_{x\to 2} f[g(x)] \neq f\left[\lim_{x\to 2} g(x)\right]$ because $\lim_{x\to 2} g(x) = 0$, and f is not cont at 2.

III. $\lim_{x\to 3} f[g(x)] \neq f\left[\lim_{x\to 3} g(x)\right]$ because $\lim_{x\to 3} g(x)$ doesn't exist.

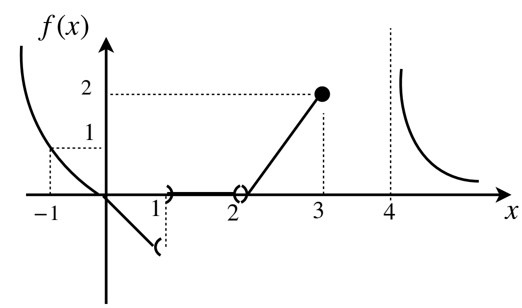
a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

Solution: b

II. False.

$\lim_{x\to 2} f[g(x)] \neq f\left[\lim_{x\to 2} g(x)\right]$ because f is not continuous at 0.

10. Given the graph:



a) $\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1 \Rightarrow \nexists \lim_{x\to -1} f(x)$
b) $\lim_{x\to 1^-} f(x) = -1$ and $\lim_{x\to 1^+} f(x) = 0 \Rightarrow \nexists \lim_{x\to 1} f(x)$
c) $\lim_{x\to 2^-} f(x) = 0$ and $\lim_{x\to 2^+} f(x) = 0 \Rightarrow \nexists \lim_{x\to 2} f(x)$
d) $\lim_{x\to 3^-} f(x) = 2$ and $\lim_{x\to 3^+} f(x) = 2 \Rightarrow \lim_{x\to 3} f(x) = 2$
e) $\lim_{x\to 4^-} f(x) = \infty$ and $\lim_{x\to 4^+} f(x) = \infty \Rightarrow \lim_{x\to 4} f(x) = \infty$

Solution: b

The correct limits for each alternative are:

$\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1 \Rightarrow \lim_{x\to -1} f(x) = 1$

$\lim_{x\to 1^-} f(x) = -1$ and $\lim_{x\to 1^+} f(x) = 0 \Rightarrow \nexists \lim_{x\to 1} f(x)$
$\lim_{x\to 2^-} f(x) = 0$ and $\lim_{x\to 2^+} f(x) = 0 \Rightarrow \lim_{x\to 2} f(x) = 0$

$\lim_{x\to 3^-} f(x) = 2$ and $\nexists \lim_{x\to 3^+} f(x) \Rightarrow \nexists \lim_{x\to 3} f(x)$
$\nexists \lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x) = \infty \Rightarrow \nexists \lim_{x\to 4} f(x)$.

11. Let $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$, and c = constant. Then:

a) All alternatives are correct.
b) $\lim_{x\to a} [c \cdot f(x)] = c \cdot \lim_{x\to a} f(x)$
c) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L + M$
d) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L - M$
e) $\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = L \cdot M$.

Solution: a

12. Solve the following limit:

$$\lim_{x\to 1} \left(x^3 - 3x\right)$$

a) -3
b) -2
c) -1
d) 0
e) 1
f) None of the above.

Solution: b

$$\lim_{x\to 1} \left(x^3 - 3x\right) = [(1)^3 - 3(1)] = -2.$$

13. Solve the following limit:

$$\lim_{x\to 4} \frac{x^2 - 16}{x - 4}$$

a) 4
b) 6
c) 8
d) 10
e) None of the above.

Solution: c

$$\lim_{x\to 4} \frac{x^2 - 16}{x - 4} = \lim_{x\to 4} \frac{(x+4)(x-4)}{(x-4)} = \lim_{x\to 4} (x+4) = 8.$$

14. Solve the following limit:

$$\lim_{x\to 1} \frac{x - 16}{\sqrt{x} - 4}$$

a) 4
b) 5
c) 6
d) 7
e) None of the above.

Solution: b

$$\lim_{x\to 1} \frac{x - 16}{\sqrt{x} - 4} = \lim_{x\to 1} \frac{(x-16)(\sqrt{x}+4)}{(\sqrt{x}-4)(\sqrt{x}+4)} = \lim_{x\to 1} \frac{(x-16)(\sqrt{x}+4)}{(x-16)} = \lim_{x\to 1} (\sqrt{x}+4) = 5$$

15. Solve the following limit:

$$\lim_{x\to 0} \frac{1 - \sqrt{x+1}}{x}$$

a) $-\frac{1}{8}$
b) $-\frac{1}{6}$
c) $-\frac{1}{4}$
d) $-\frac{1}{2}$
e) None of the above.

Solution: d

Let $y = \sqrt{x+1}$

Since $y = \sqrt{x+1}$ then $y^2 = x + 1 \Rightarrow x = y^2 - 1$
$x \to 0 \Rightarrow y \to 1$.

Thus:
$$\lim_{x\to 0} \frac{1 - \sqrt{x+1}}{x} = \lim_{y\to 1} \frac{1 - y}{y^2 - 1} = \lim_{y\to 1} \frac{(1-y)}{(y+1)(y-1)} = \lim_{y\to 1} \frac{-1}{y+1} = -\frac{1}{2}$$

16. Solve the following limit:

$$\lim_{x\to 5} \left[\frac{x^2 - 25}{x - 5}\right]^2$$

a) 16
b) 36
c) 64
d) 100
e) None of the above.

Solution: d

Since
then:

$$\lim_{x\to5}\left[\frac{x^2-25}{x-5}\right]^2=\left[\lim_{x\to5}\frac{x^2-25}{x-5}\right]^2=10^2=100.$$

17. Let $p(x)=a_nx^n+a_{n-1}+x^{n-1}+\ldots+a_0$ and $g(x)=b_nx^n+b_{n-1}+x^{n-1}+\ldots+b_0$ be polynomials then:

a) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to\infty}\frac{a_nx^n}{b_nx^n}$

b) $\lim_{x\to1}\frac{f(x)}{g(x)}=\lim_{x\to1}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to1}\frac{a_nx^n}{b_nx^n}$

c) $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to a}\frac{a_nx^n}{b_nx^n}$

d) $\lim_{x\to-\infty}\frac{f(x)}{g(x)}=\lim_{x\to-\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to-\infty}\frac{a_nx^n}{b_nx^n}$

e)There are more then one alternative correct.

Solution: e

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to\infty}\frac{a_nx^n}{b_nx^n}$$

$$\lim_{x\to-\infty}\frac{f(x)}{g(x)}=\lim_{x\to-\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_0}=\lim_{x\to-\infty}\frac{a_nx^n}{b_nx^n}$$

where a_nx^n and b_nx^n are the leading terms of $p(x)$ and $g(x)$.

18. Solve:

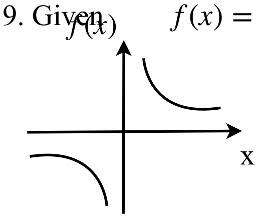
$$\lim_{x\to-\infty}\frac{x^4+3x^2-x+\pi}{x^3-5x^2+3x+e}$$

a) 0
b) 1
c) ∞
d) $-\infty$
e) None of the above.

Solution: d

$$\lim_{x\to-\infty}\frac{x^4+3x^2-x+\pi}{x^3-5x^2+3x+e}=\lim_{x\to-\infty}\frac{x^4}{x^3}=\lim_{x\to-\infty}x=-\infty$$

19. Given $f(x)=\frac{3}{x}$:



Then:
a) $\lim_{x\to\infty}f(x)=0$
b) $\lim_{x\to-\infty}f(x)=0$
c) All alternatives are correct.
d) $\lim_{x\to0^-}f(x)=-\infty$
e) $\lim_{x\to0^+}f(x)=\infty$

Solution: c

20. Let $f(x)=e^x$ then:

a) $\lim_{x\to\infty}f(x)=0$
b) $\lim_{x\to-\infty}f(x)=1$
c) $\lim_{x\to0}f(x)=1$
d) $\lim_{x\to1}f(x)=\infty$
e) All alternatives are correct.

Solution: c

a) False. $\lim_{x\to\infty}e^x=e^\infty=\infty.$

b) False. $\lim_{x\to-\infty}e^x=e^{-\infty}=\frac{1}{e^\infty}=0.$

c) True. $\lim_{x\to0}e^x=e^0=1.$

d) False. $\lim_{x\to1}e^x=e^1=e.$

Name:______________________________ **Age:**_____ **Id:**________________ **Course:**__________

PART 2: SOLUTIONS

Consulting

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Solve:

$$\lim_{x\to 1} \frac{x^2+x-2}{x-1}$$

Solution:

$$\lim_{x\to 1} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} \frac{(x+2)(x-1)}{x-1} = \lim_{x\to 1} x+2 = 3$$

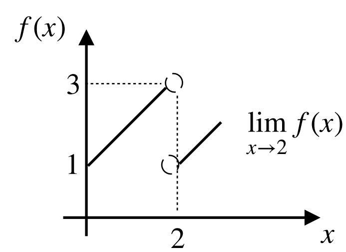
22. Solve:

$$\lim_{x\to -\infty} \frac{6x^2+x-25}{3x^2-4x-5}$$

Solution:

$$\lim_{x\to -\infty} \frac{6x^2+x-25}{3x^2-4x-5} = \lim_{x\to -\infty} \frac{6x^2}{3x^2} = \lim_{x\to -\infty} \frac{6}{3} = 2.$$

23. Given the graph, show if the limit exist or not. (Hint: Use lateral limits)



$$\lim_{x\to 2} f(x) = ?$$

Solution:

$$\lim_{x\to 2^-} f(x) = 3 \text{ and } \lim_{x\to 2^+} f(x) = 1 \text{ then } \lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \Rightarrow \nexists \lim_{x\to 2} f(x)$$

24. Solve:

$$\lim_{x\to 1} \frac{2x+3}{6-x}$$

Solution:

$$\lim_{x\to 1} \frac{2x+3}{6-x} = \frac{2(1)+3}{6-1} = 1.$$

25. Solve:

$$\lim_{x \to 0} \frac{\sqrt{x+9} - \sqrt{9-x}}{x}$$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{x+9} - \sqrt{9-x}}{x} = \lim_{x \to 0} \frac{\left(\sqrt{x+9} - \sqrt{9-x}\right)\left(\sqrt{x+9} + \sqrt{9-x}\right)}{x\left(\sqrt{x+9} + \sqrt{9-x}\right)}$$

$$= \lim_{x \to 0} \frac{2}{\left(\sqrt{x+9} + \sqrt{9-x}\right)} = \frac{1}{\sqrt{9}} = \frac{1}{3}.$$

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