PART 1: QUESTIONS

Name:______________________________ **Age:**_____ **Id:**______________ **Course:**__________

Trigonometry - Exam 1 **Lessons: 20 - 22**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. Given:

I. Trigonometry is the branch of mathematics that deals with the relationship between the sides and angles of triangles and the study of trigonometric functions.
II. The radian is the standard unit of angular measure that is equal to the length of a corresponding arc of a unit circle.
III. The number π is a mathematical constant defined as the ratio of a circle's circumference to its diameter.

Then,

a) I, II, and III are correct.
b) I, II, and III are **incorrect**.
c) Only I and II are correct.
d) Only I and III are correct.
e) Only II and III are correct.

Solution: a

I. True. Definition of trigonometry.
II. True. Definition of radian.
III. True. The number π is a mathematical constant (approximately equal to 3.14159) defined as the ratio of a circle's circumference to its diameter. Today, π has various equivalent definitions and appears in many formulas in all areas of mathematics and physics.
Thus, I, II, and III are correct.

2. What is the measure in degrees of the angle $\frac{5\pi}{3}$?

a) 150^o b) 210^o c) 330^o d) 390^o e) None of the above.

Solution: e

$$A = \frac{5\pi}{3} = \frac{5(180^o)}{3} = 300^o$$

3. What is the measure in degrees of the angle $-\frac{5\pi}{6}$?

a) 150^o b) 210^o c) 330^o d) 390^o e) None of the above.

Solution: b

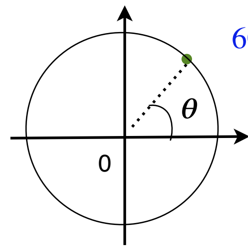
$$A = -\frac{5\pi}{6} = \frac{7\pi}{6} = \frac{7(180^o)}{6} = 210^o$$

4. In the trigonometric ball, $\theta = 60^o$ is:

a) $\frac{\pi}{3}$ b) $\frac{7\pi}{3}$ c) All alternatives are correct.

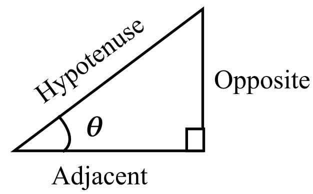
d) $-\frac{5\pi}{3}$ e) -300^o

Solution: c



$$60^o, \frac{\pi}{3}, \frac{7\pi}{3}, -\frac{5\pi}{3}, -300^o, ..$$

5. Given:



Then,

I. $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$

II. $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

III. $\csc(\theta) = \frac{1}{\cos(\theta)}$

a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

Solution: a

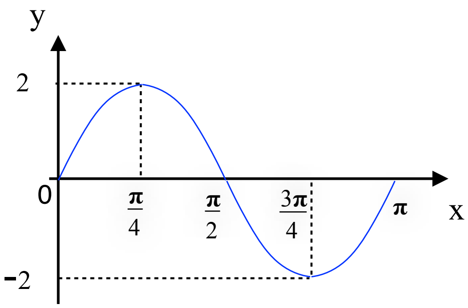
I. True: $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$

II. True: $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

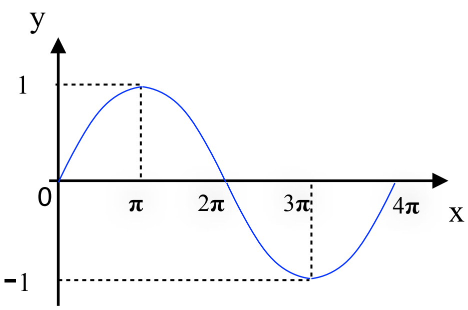
III. False: $\csc(\theta) = \frac{1}{\sin(\theta)}$

6. The graph of $y = 2\sin(\frac{x}{2})$ is:

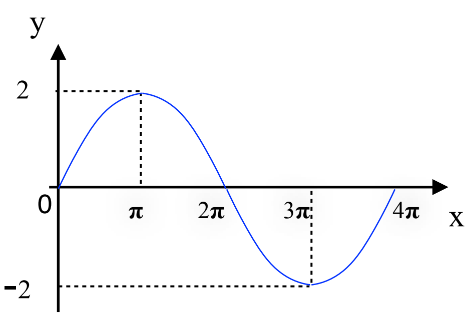
a)



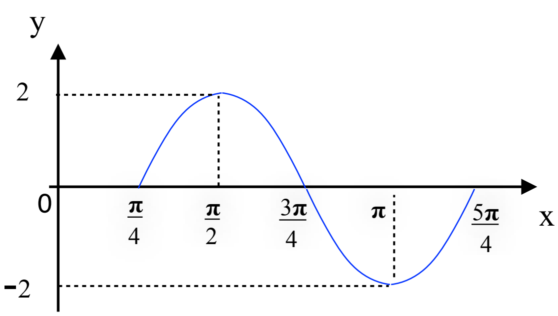
b)



c)



d)



e) None of the above.

Solution: c

Let $y = A\sin(Bx - C)$

Since $y = 2\sin(\frac{x}{2})$ then:

Amplitude: A $= 2$

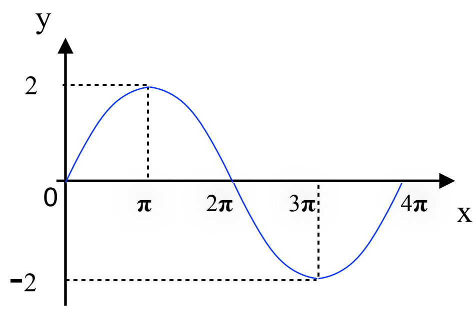
Start Point: $Bx - C = 0 \Rightarrow x = \frac{C}{B}$

$\frac{x}{2} = 0 \Rightarrow x = 0$

Period: $\frac{2\pi}{B}$

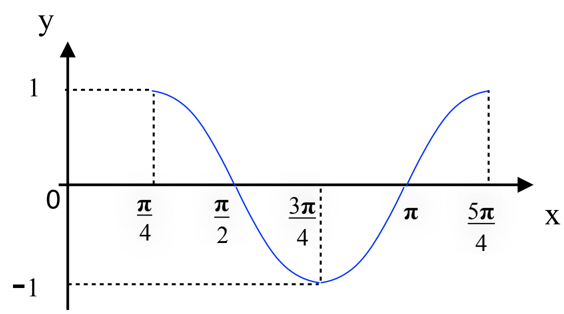
$P = \frac{2\pi}{1/2} = 4\pi$

Thus,

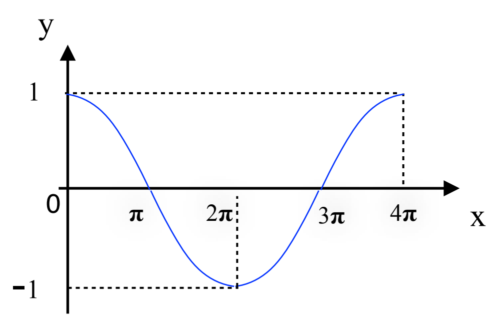


7. The graph of $y = 2\cos(\frac{x}{2})$ is:

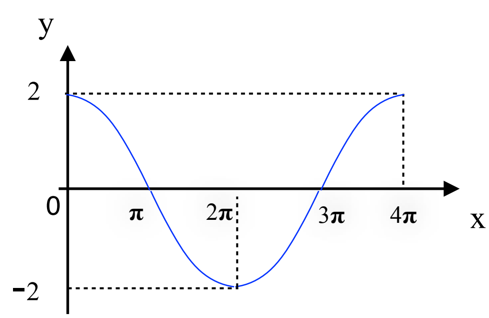
a)



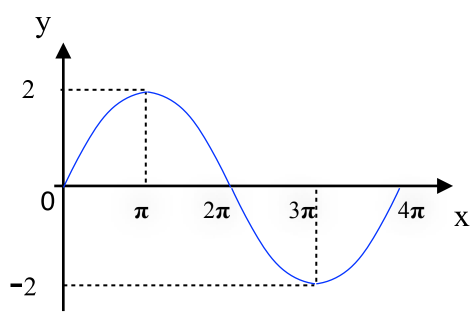
b)



c)



d)



e) None of the above.

Solution: c

Let $y = A\cos(Bx - C)$

Since $y = 2\cos(\frac{x}{2})$ then:

Amplitude: $A = 2$

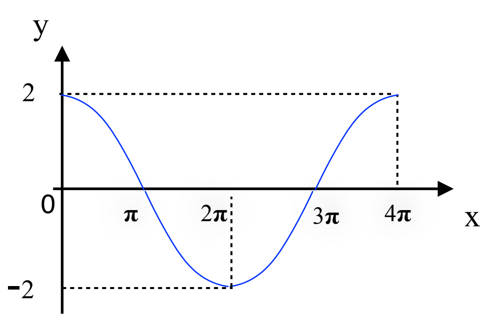
Start Point: $Bx - C = 0 \Rightarrow x = \frac{C}{B}$

$\frac{x}{2} = 0 \Rightarrow x = 0$

Period: $\frac{2\pi}{B}$

$P = \frac{2\pi}{1/2} = 4\pi$

Thus,



8. Given angle $\theta_1 = \frac{\pi}{12}$ in the I Quadrant, the correspondent angles $\theta_2, \theta_3, \theta_4$ in the quadrants II, III, and IV are:

a) $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, and $\frac{5\pi}{3}$

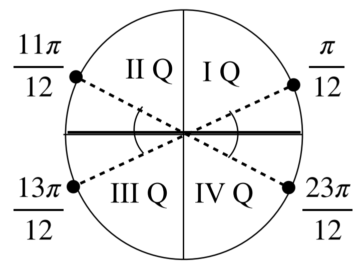
b) $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$

c) $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$

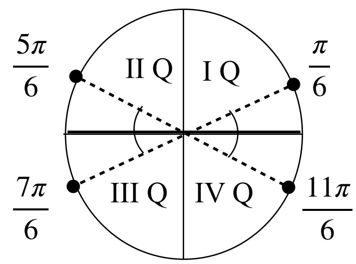
d) $\frac{11\pi}{12}$, $\frac{13\pi}{12}$, and $\frac{23\pi}{12}$

e) None of the above.

Solution: d



9. Given:



The general solution is:

I. $\left.\begin{aligned} x &= \frac{\pi}{6} + 2\pi k \\ x &= \frac{5\pi}{6} + 2\pi k \\ x &= \frac{7\pi}{6} + 2\pi k \\ x &= \frac{11\pi}{6} + 2\pi k \end{aligned}\right\} k \in \mathbb{Z}$

II. $\left.\begin{aligned} x &= \frac{\pi}{6} + \pi k \\ x &= \frac{5\pi}{6} + \pi k \end{aligned}\right\} k \in \mathbb{R}$

III. $x = \pm\frac{\pi}{6} + \pi k, k \in \mathbb{Z}$

a) I, II, and III are incorrect.
b) I, II, and III are correct.
c) Only I and II correct.
d) Only I and III are correct.
e) None of the above.

Solution: b

10. Solve: $\sin x = -\dfrac{\sqrt{3}}{2}$, where $0 \le x < 2\pi$

a) $x = \dfrac{\pi}{6}$ or $x = \dfrac{5\pi}{6}$

b) $x = \dfrac{\pi}{4}$ or $x = \dfrac{3\pi}{4}$

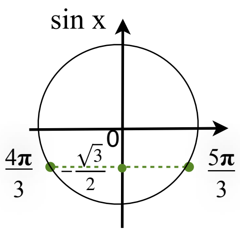
c) $x = \dfrac{7\pi}{6}$ or $x = \dfrac{11\pi}{6}$

d) $x = \dfrac{5\pi}{4}$ or $x = \dfrac{7\pi}{4}$

e) None of the above.

Solution: e

$\sin x = -\dfrac{\sqrt{3}}{2}$



Thus, $x = \dfrac{4\pi}{3}$ or $x = \dfrac{5\pi}{3}$

11. Solve: $\cos x = \dfrac{1}{2}$, where $0 \le x < 2\pi$

a) $x = \dfrac{\pi}{3}$ or $x = \dfrac{5\pi}{3}$

b) $x = \dfrac{\pi}{4}$ or $x = \dfrac{7\pi}{4}$

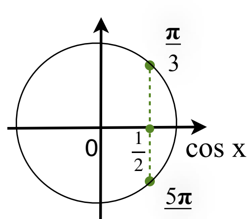
c) $x = \dfrac{2\pi}{3}$ or $x = \dfrac{4\pi}{3}$

d) $x = \dfrac{3\pi}{4}$ or $x = \dfrac{5\pi}{4}$

e) None of the above.

Solution: a

$\cos x = \dfrac{1}{2}$



Thus, $x = \dfrac{\pi}{3}$ or $x = \dfrac{5\pi}{3}$

12. Solve: $\tan x = -\dfrac{\sqrt{3}}{3}$, where $0 \le x < 2\pi$

a) $x = \dfrac{\pi}{6}$ or $x = \dfrac{7\pi}{6}$

b) $x = \dfrac{\pi}{4}$ or $x = \dfrac{5\pi}{4}$

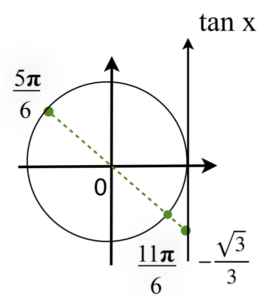
c) $x = \dfrac{2\pi}{3}$ or $x = \dfrac{5\pi}{3}$

d) $x = \dfrac{3\pi}{4}$ or $x = \dfrac{7\pi}{4}$

e) None of the above.

Solution: e

$\tan x = -\dfrac{\sqrt{3}}{3}$



Thus, $x = \dfrac{5\pi}{6}$ or $x = \dfrac{11\pi}{6}$

13. Solve: $\sin x = \dfrac{\sqrt{2}}{2}$

a) $x = \dfrac{\pi}{6} + 2\pi k$ or $x = \dfrac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

b) $x = \dfrac{\pi}{4} + 2\pi k$ or $x = \dfrac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$

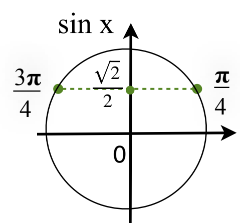
c) $x = \dfrac{7\pi}{6} + 2\pi k$ or $x = \dfrac{11\pi}{6} + 2\pi k, k \in \mathbb{Z}$

d) $x = \dfrac{5\pi}{4} + 2\pi k$ or $x = \dfrac{7\pi}{4} + 2\pi k, k \in \mathbb{Z}$

e) None of the above.

Solution: b

$\sin x = \dfrac{\sqrt{2}}{2}$



Thus, $x = \dfrac{\pi}{4} + 2\pi k$ or $x = \dfrac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$

14. Solve: $\cos x = -\dfrac{\sqrt{2}}{2}$

a) $x = \dfrac{\pi}{3} + 2\pi k$ or $x = \dfrac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}$

b) $x = \dfrac{\pi}{4} + 2\pi k$ or $x = \dfrac{7\pi}{4} + 2\pi k, k \in \mathbb{Z}$

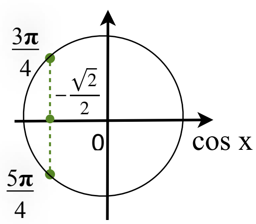
c) $x = \dfrac{2\pi}{3} + 2\pi k$ or $x = \dfrac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$

d) $x = \dfrac{3\pi}{4} + 2\pi k$ or $x = \dfrac{5\pi}{4} + 2\pi k, k \in \mathbb{Z}$

e) None of the above.

Solution: d

$\cos x = -\dfrac{\sqrt{2}}{2}$



Thus, $x = \dfrac{3\pi}{4} + 2\pi k$ or $x = \dfrac{5\pi}{4} + 2\pi k, k \in \mathbb{Z}$

15. Solve: $\tan x = -\sqrt{3}$

a) $x = \dfrac{\pi}{6} + 2\pi k$ or $x = \dfrac{7\pi}{6} + 2\pi k, k \in \mathbb{Z}$

b) $x = \dfrac{\pi}{4} + 2\pi k$ or $x = \dfrac{5\pi}{4} + 2\pi k, k \in \mathbb{Z}$

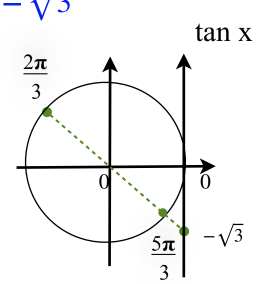
c) $x = \dfrac{2\pi}{3} + 2\pi k$ or $x = \dfrac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}$

d) $x = \dfrac{3\pi}{4} + 2\pi k$ or $x = \dfrac{7\pi}{4} + 2\pi k, k \in \mathbb{Z}$

e) None of the above.

Solution: c

$\tan x = -\sqrt{3}$



Thus, $x = \dfrac{2\pi}{3} + 2\pi k$ or $x = \dfrac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}$

16. Solve: $\csc^2 x = 2$

a) $x = \dfrac{\pi}{6} + \pi k$ or $x = \dfrac{5\pi}{6} + \pi k, k \in \mathbb{Z}$

b) $x = \dfrac{\pi}{3} + \pi k$ or $x = \dfrac{2\pi}{3} + \pi k, k \in \mathbb{Z}$

c) $x = \dfrac{\pi}{4} + 2\pi k$ or $x = \dfrac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$

d) $x = \dfrac{\pi}{2} + 2\pi k$ or $x = \dfrac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

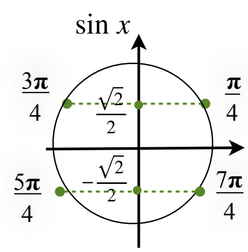
e) None of the above.

Solution: c

$\csc^2 x = 2$

$\left(\dfrac{1}{\sin x}\right)^2 x = 2$

$\sin x = \pm\dfrac{\sqrt{2}}{2}$



Thus, $x = \dfrac{\pi}{4} + \pi k$ or $x = \dfrac{3\pi}{4} + \pi k, k \in \mathbb{Z}$

17. Solve: $\tan^2 x + \tan x = 0$

a) $x = \pi k$ or $x = \dfrac{7\pi}{6} + 2\pi k$ or $x = \dfrac{11\pi}{6} + 2\pi k, k \in \mathbb{Z}$

b) $x = \dfrac{\pi}{2} + \pi k$ or $x = \dfrac{2\pi}{3} + 2\pi k$ or $x = \dfrac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$

c) $x = \pi k$ or $x = \dfrac{3\pi}{4} + \pi k, k \in \mathbb{Z}$

d) $x = \pi k$ or $x = \dfrac{5\pi}{4} + \pi k, k \in \mathbb{Z}$

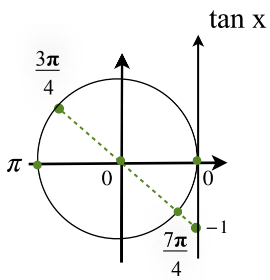
e) None of the above.

Solution: c

$\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ or $\tan x = -1$



Thus, $x = \pi k$ or $x = \dfrac{3\pi}{4} + \pi k, k \in \mathbb{Z}$

18. Solve the trigonometric inequality:

$$\cos x \le -\frac{\sqrt{3}}{2}$$

a) $S = \left\{x \in \mathbb{R} / \frac{\pi}{4} + 2\pi k \le x \le \frac{3\pi}{4} + 2\pi k, \quad k \in \mathbb{Z}\right\}$

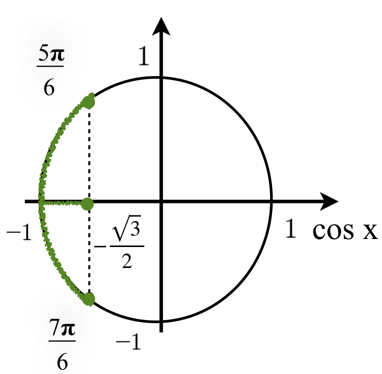
b) $S = \left\{x \in \mathbb{R} / \frac{5\pi}{6} + 2\pi k \le x \le \frac{7\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}\right\}$

c) $S = \left\{x \in \mathbb{R} / \frac{\pi}{6} + \pi k \le x < \frac{\pi}{2} + \pi k, \quad k \in \mathbb{Z}\right\}$

d) $S = \left\{x \in \mathbb{R} / \frac{\pi}{3} + \pi k \le x < \frac{\pi}{2} + \pi k, \quad k \in \mathbb{Z}\right\}$

e) None of the above.

Solution: b



Thus,
$S = \left\{x \in \mathbb{R} / \frac{5\pi}{6} + 2\pi k \le x \le \frac{7\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}\right\}$

19. Solve the trigonometric inequality:

$$\cos x > -\frac{1}{2}$$

a) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{3} + 2\pi k \text{ or } \frac{5\pi}{3} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

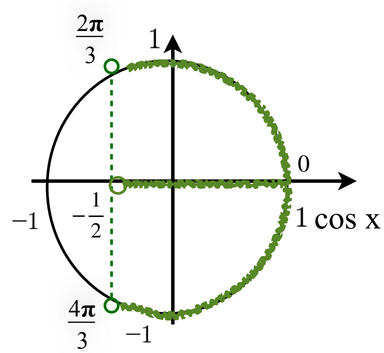
b) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{4} + 2\pi k \text{ or } \frac{7\pi}{4} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

c) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{6} + 2\pi k \text{ or } \frac{7\pi}{6} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

d) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{2} + 2\pi k \text{ or } \frac{3\pi}{2} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

e) None of the above.

Solution: e



Thus,
$S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{2\pi}{3} + 2\pi k \text{ or } \frac{4\pi}{3} + 2\pi k < x < 2\pi k, \quad k \in \mathbb{Z}\right\}$

20. Solve the trigonometric inequality:

$$\cos x > \frac{4}{3}$$

a) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{6} + 2\pi k \text{ or } \frac{11\pi}{6} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

b) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{3} + 2\pi k \text{ or } \frac{5\pi}{3} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

c) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{4} + 2\pi k \text{ or } \frac{7\pi}{4} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

d) $S = \left\{x \in \mathbb{R} / 2\pi k \le x < \frac{\pi}{2} + 2\pi k \text{ or } \frac{3\pi}{2} + 2\pi k < x < 2\pi k, k \in \mathbb{Z}\right\}$

e) $S = \varnothing$

Solution: e

$-1 \le \cos x \le 1 \Rightarrow \nexists x \in \mathbb{R}$

Thus, $S = \varnothing$.

Name:____________________ **Age:**_____ **Id:**____________ **Course:**__________

PART 2: SOLUTIONS

Consulting

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
4					
5					
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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. In trigonometry, you can do the following approach:

If $\theta < 5^o$ (degree) then $\theta \simeq \sin\theta$ (almost the same). (True)

Then a student concludes:

If $\theta = 2^o$ (degree) then $\sin\theta \simeq 2$. (False)

Explain why the student's conclusion is wrong since $-1 \leq \sin\theta \leq 1$.

Solution:

If $\theta < 5^o$ (degree) then $\theta \simeq \sin\theta$ (almost the same) is true if θ is measure in radians.

22. Prove:

$$\frac{(1+\sin x)(1-\sin x)}{\cos^2 x} = 1$$

Solution:

$$\frac{(1+\sin x)(1-\sin x)}{\cos^2 x} = \frac{(1-\sin^2 x)}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$$

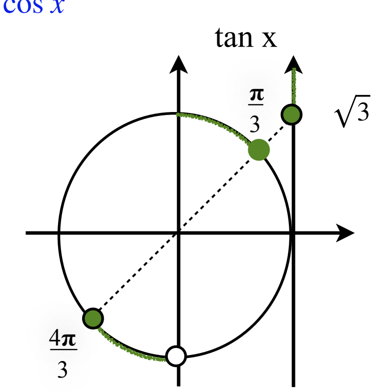
23. Solve: $\sin x = \sqrt{3}\cos x$, where $0 \leq x < 2\pi$.

Solution:

Note that if $\cos x = 0$ then $\sin x \neq 0$ (see trigonometric ball) $\Rightarrow 0 \neq 0$ false.

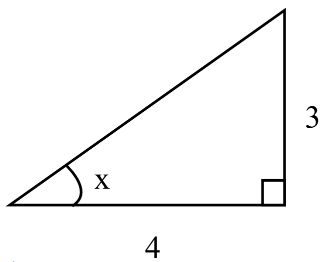
$\sin x = \sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \sqrt{3} \Rightarrow \tan x = \sqrt{3}$



Thus, $S = \{\frac{\pi}{3}, \frac{4\pi}{3}\}$

24. Find $\tan x + \cot x$:



Solution:

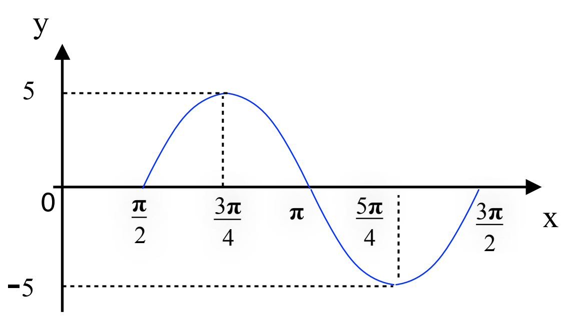
Let $(b = 3 \text{ and } c = 4) \Rightarrow a = \sqrt{(b^2 + c^2)} = 5$

$\sin x = \frac{b}{a} = \frac{3}{5}$ and $\cos x = \frac{c}{a} = \frac{4}{5}$

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$

$\tan x + \cot x = \frac{1}{(3/5)(4/5)} = \frac{25}{12}$

25. Find the function of the following graph:



Solution:

$y = A \sin(Bx - C)$

Amplitude: $A = 5$

Period: $P = \frac{2\pi}{B} = \pi \Rightarrow B = 2$

Start point: $\frac{C}{B} = \frac{\pi}{2} \Rightarrow \frac{C}{2} = \frac{\pi}{2} \Rightarrow C = \pi$

Thus, $y = 5\sin(2x - \pi)$.